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MULTIPOINT GLOBAL OPTIMAL SHAPE DESIGN BY MORPHING

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Abstract. *The determination of the global optimized (GO) shape of a flying configuration (FC), which is of minimum drag at two cruising Mach numbers, can be obtained by morphing. Two movable leading edge flaps are used for this purpose. They are retracted at higher Mach number and stretched by the lower one. Two consecutive enlarged variational problems with free boundaries occur and are solved by using the own iterative optimum-optimorum theory as optimization strategy. It uses only in its first step of iteration hyperbolic potential solutions and determine the inviscid GO shape of FC as surrogate model. Up the second step of iteration own hybrid solutions for Navier-Stokes layer are used and the total drag is the new functional.*

1 INTRODUCTION

The aim of this study is to determine the aerodynamic, global optimized (GO) shape of a flying configuration (FC), which is flying at two supersonic cruising Mach numbers. Due to the fact that the GO shape of a FC, with respect to minimum drag at cruise, changes its optimal planform very much, if the supersonic cruising Mach number is varied, this aim can be correct realized only by morphing. The morphing is obtained by using movable leading edge flaps. The surfaces of the wing, of the fuselage and of the flaps are supposed to be expressed or approximated in form of different superpositions of homogeneous polynoms in two variables with free coefficients. These coefficients, together with the similarity parameters of the planforms of these components of FC, are the free parameters of optimization. The determination of the GO shape of FC at two supersonic cruising Mach numbers leads to the solving of two consecutive extended variational problems with free boundaries. The first one consists in the determination of the GO shape of the unmovable integrated wing-fuselage part of FC with retracted flaps, which is of minimum drag at higher cruising Mach number. The second one consists in the determination of the shape of the stretched leading edge flaps, in such a manner that the entire FC with stretched flaps is of minimum drag, at the second lower cruising Mach number.

An own developed mathematical strategy called iterative optimum-optimorum (OO) theory is used for the solving of these both consecutive variational problems. In the first step of the iterative OO theory, the surrogate GO shape is determined by using the three-dimensional hyperbolic potential solutions of the author as start solutions, as in [1] and the OO theory as strategy of optimization of the inviscid drag functional. It follows a computational checking of the inviscid GO shape of the surrogate model by using hybrid solutions for the Navier-Stokes layer (NSL), proposed by the author, as in [1,2] . The friction drag coefficient is computed and a weak interaction aerodynamics-structure can be performed. Up the second step of iteration the new drag functional is the total drag and additional constraints due to the structure requests can occur. The following premises for the optimization are taken into consideration:

- the FCs have sharp leading edges in order to avoid the bow shock and to fly with characteristic surface;
- the junction lines wing-fuselage and wing-flaps begin at the apex of the wing, in order to avoid the sonic boom interference;
- the surface of FC must be integrated (namely the wing and the fuselage have the same tangent plane along the junction lines between the wing and the fuselage) in order to avoid the negative effects of corners;
- the leading edges conturnements of their subsonic leading edges of wing with flaps in retracted position and of subsonic flaps in stretched position must be reduced. For these purposes the Kutta condition on leading edges is used.

2 THREE-DIMENSIONAL HYPERBOLIC START SOLUTIONS FOR INVISCID GLOBAL OPTIMAL SHAPE DESIGN OF FLYING CONFIGURATION

Let us firstly consider an integrated wing-fuselage FC with arbitrary camber, twist and thickness distributions, which is flying at the higher cruising Mach number M_∞ with the flaps in retracted position. The FC with retracted flaps is considered like an integrated wing-fuselage FC with arbitrary camber, twist and thickness distributions and is flying at the higher cruising Mach number M_∞. Dimensionless coordinates are used for the computation of the distributions of velocity's components:

$$\tilde{x}_1 = \frac{x_1}{h_1} \quad , \quad \tilde{x}_2 = \frac{x_2}{\ell_1} \quad , \quad \tilde{x}_3 = \frac{x_3}{h_1} \, . \qquad (\, \tilde{y} = \frac{\tilde{x}_2}{\tilde{x}_1} \,) \tag{1}$$

Further, integrated wing-fuselage FCs are considered, namely, for which the mean surface is continuous and the thickness distributions on the wing and on the fuselage are different and three-dimensional hyperbolic potential solutions, previously given by the author as in [1], are used as start solutions for the determination of the inviscid GO shape of FC with retracted flaps, at higher cruising Mach number M_∞ and also for the determination of the inviscid GO shape of the stretched flaps of the FC, at lower cruising Mach number M'_∞ .

The downwashes on the thin and thick-symmetrical components of the integrated FCs, with flaps in retracted position w and w^*, w'^* (on the wing and on the fuselage of FCs), are expressed in form of superposition of homogeneous polynoms with arbitrary coefficients:

$$w \equiv \widetilde{w} = \sum_{m=1}^{N} \widetilde{x}_1^{m-1} \sum_{k=0}^{m-1} \widetilde{w}_{m-k-1,k} \left|\widetilde{y}\right|^k \quad ,$$

$$w^* \equiv \widetilde{w}^* = \sum_{m=1}^{N} \widetilde{x}_1^{m-1} \sum_{k=0}^{m-1} \widetilde{w}^*_{m-k-1,k} \left|\widetilde{y}\right|^k \quad , \qquad w'^* \equiv \overline{w}^* = \sum_{m=1}^{N} \widetilde{x}_1^{m-1} \sum_{k=0}^{m-1} \overline{w}^*_{m-k-1,k} \left|\widetilde{y}\right|^k \quad . \tag{2a-c}$$

The coefficients of the downwashes and the similarity parameter $\nu = B\ \ell \quad \bar{\nu} = Bc$ ($\ell = \ell_1 / h_1$, $B = \sqrt{M_\infty^2 - 1}$), of the planform of the wing are the free parameters of the optimization and $\ell,\ \ell_1,\ h_1$ are the dimensionless span, the half-span and the depth of the planform of the delta wing. The quotient $\bar{k} \equiv \bar{\nu} / \nu = c / \ell$ of the similarity parameters of the wing and of the fuselage, which depend on the purpose of the FC, is supposed constant. If the principle of minimal singularities (which fulfill the jumps of the velocity's components) and the hydrodynamic analogy of Carafoli are used, the following expressions for the axial disturbances of the thin and of the thick-symmetrical components of the integrated wing-fuselage FC with subsonic leading edges are obtained, as in [1]:

$$u \equiv \ell\ \widetilde{u} = \ell \sum_{n=1}^{N} \widetilde{x}_1^{n-1} \left\{ \sum_{q=0}^{E\left(\frac{n}{2}\right)} \frac{\widetilde{A}_{n,2q} \widetilde{y}^{2q}}{\sqrt{1-\widetilde{y}^2}} + \sum_{q=1}^{E\left(\frac{n-1}{2}\right)} \widetilde{C}_{n,2q} \widetilde{y}^{2q} \cosh^{-1} \sqrt{\frac{1}{\widetilde{y}^2}} \right\} \quad , \tag{3a}$$

$$u^* \equiv \ell\ \widetilde{u}^* = \ell \sum_{n=1}^{N} \widetilde{x}_1^{n-1} \left\{ \sum_{q=0}^{n-1} \widetilde{H}^*_{nq} \widetilde{y}^q \left(\cosh^{-1} M_1 + (-1)^q \cosh^{-1} M_2 \right) \right.$$

$$+ \sum_{q=0}^{n-1} \widetilde{G}^*_{nq}\ \widetilde{y}^q \left(\cosh^{-1} S_1 + (-1)^q \cosh^{-1} S_2 \right)$$

$$\left. + \sum_{q=1}^{E\left(\frac{n-1}{2}\right)} \widetilde{C}^*_{n,2q}\ \widetilde{y}^{2q} \cosh^{-1} \sqrt{\frac{1}{\nu^2 \widetilde{y}^2}} + \sum_{q=0}^{E\left(\frac{n-2}{2}\right)} \widetilde{D}^*_{n,2q}\ \widetilde{y}^{2q} \sqrt{1-\nu^2 \widetilde{y}^2} \right\} \quad . \tag{3b}$$

$$\left(M_{1,2} = \sqrt{\frac{(1+\nu)(1 \mp \nu\ \widetilde{y})}{2\nu(1 \mp \widetilde{y})}} \right)$$

The lift and the pitching moment coefficients of the integrated wing-fuselage FCs with stretched flaps, computed with the hyperbolic potential theory, are the following:

$$C_\ell \equiv 8\ell \int_{\widetilde{O}\widetilde{A}_1\widetilde{C}_1} \widetilde{u}\ \widetilde{x}_1 d\widetilde{x}_1 d\widetilde{y} \quad , \qquad C_m \equiv -8\ell \int_{\widetilde{O}\widetilde{A}_1\widetilde{C}_1} \widetilde{u}\ \widetilde{x}_1^2 d\widetilde{x}_1 d\widetilde{y} \quad . \tag{4a,b}$$

The inviscid drag coefficients of the thin, thick-symmetrical and thick, lifting FCs with retracted flaps are quadratical forms with respect to the downwashes coefficients :

$$C_d \equiv \ell \widetilde{C}_d = 8\ell \int\limits_{\widetilde{O}\widetilde{A}_1\widetilde{C}} \widetilde{u}\ \widetilde{w}\ \widetilde{x}_1 d\widetilde{x}_1 d\widetilde{y}\ ,$$

$$C_d^* \equiv \ell \widetilde{C}_d^* = 8\ell\ \Big[\int\limits_{\widetilde{O}\widetilde{C}_1\widetilde{C}} \widetilde{u}^* \overline{w}^* \widetilde{x}_1 d\widetilde{x}_1 d\widetilde{y} + \int\limits_{\widetilde{O}\widetilde{A}_1\widetilde{C}_1} \widetilde{u}^* \widetilde{w}^* \widetilde{x}_1 d\widetilde{x}_1 d\widetilde{y}\ \Big],$$

$$C_d^{(i)} \equiv \ell \widetilde{C}_d^{(i)} = \ell \left(\widetilde{C}_d + \widetilde{C}_d^* \right). \tag{5a-c}$$

Let us now consider the FCs with stretched flaps. The downwashes w and w^*, $\overline{w}^*$ on the thin and thick-symmetrical components of the integrated FC remain unchanged because the surface of the wing part of FC remains unchanged. The downwashes w'' and w''^* on the thin and thick-symmetrical stretched flaps are also supposed to be approximated in form of superposition of homogeneous polynoms in two variables with free coefficients, namely:

$$w'' \equiv \widetilde{w}' = \sum_{m=1}^{N} \widetilde{x}_1^{m-1} \sum_{k=0}^{m-1} \widetilde{w}'_{m-k-1,k} \left|\widetilde{y}\right|^k \quad , \quad w''^* \equiv \widetilde{w}'^* = \sum_{m=1}^{N} \widetilde{x}_1^{m-1} \sum_{k=0}^{m-1} \widetilde{w}'^*_{m-k-1,k} \left|\widetilde{y}\right|^k\ . \tag{6a,b}$$

The corresponding axial disturbance velocities of the thin and thick-symmetrical components of the integrated FCs with stretched flaps at the second lower cruising Mach number M'_∞ are, as follows:

$$u = L \sum_{n=1}^{N} \widetilde{x}_1^{n-1} \left[\sum_{q=0}^{n-1} \widetilde{\widetilde{A}}_{nq} \widetilde{y}^q \left(\cosh^{-1} N_1'' + (-1)^q \cosh^{-1} N_2'' \right) + \sum_{q=0}^{E\left(\frac{n}{2}\right)} \frac{\widetilde{\widetilde{F}}_{n,2q}\ \widetilde{y}^{2q}}{\sqrt{1-\widetilde{y}^2}} \right.$$

$$\left. + \sum_{q=1}^{E\left(\frac{n-1}{2}\right)} \widetilde{\widetilde{C}}_{n,2q} \widetilde{y}^{2q} \cosh^{-1} \sqrt{\frac{1}{\widetilde{y}^2}}\ \right]\ . \tag{7a}$$

$$\left(\quad N'_{1,2} = \sqrt{\frac{(1+k')\ (1\mp\widetilde{y})}{2\ (k'\mp\widetilde{y})}} \quad \right)$$

$$u^* = L \sum_{n=1}^{N} \widetilde{x}_1^{n-1} \left[\sum_{q=0}^{n-1} \widetilde{\widetilde{H}}^*_{nq} \, \widetilde{y}^q \left(\cosh^{-1} M_1' + (-1)^q \cosh^{-1} M_2' \right) \right.$$

$$+ \sum_{q=0}^{E\left(\frac{n}{2}\right)} \widetilde{\widetilde{D}}^*_{n,2q} \, \widetilde{y}^{2q} \sqrt{1-\widetilde{\nu}'^2 \, \widetilde{y}^2} + \sum_{q=1}^{E\left(\frac{n-1}{2}\right)} \widetilde{\widetilde{C}}^*_{n,2q} \, \widetilde{y}^{2q} \cosh^{-1} \sqrt{\frac{1}{\widetilde{\nu}'^2 \, \widetilde{y}^2}}$$

$$+ \sum_{q=0}^{n-1} \widetilde{\widetilde{F}}^*_{nq} \, \widetilde{y}^q \left(\cosh^{-1} N_1'' + (-1)^q \cosh^{-1} N_2'' \right)$$

$$\left. + \sum_{q=0}^{n-1} \widetilde{\widetilde{G}}^*_{nq} \, \widetilde{y}^q \left(\cosh^{-1} S_1' + (-1)^q \cosh^{-1} S_2' \right) \right] \quad . \qquad (7b)$$

$$\left(M'_{1,2} = \sqrt{\frac{(1+\nu')\,(1\mp\widetilde{\nu}'\widetilde{y})}{2(\nu' \mp \widetilde{\nu}'\widetilde{y})}} \; , \quad N''_{1,2} = \sqrt{\frac{(1+\widetilde{\nu}')\,(1\mp\widetilde{\nu}'\widetilde{y})}{2\nu'(1\mp\widetilde{y})}} \; , \quad S'_{1,2} = \sqrt{\frac{(1+\overline{\nu}')\,(1\mp\widetilde{\nu}'\widetilde{y})}{2(\overline{\nu}' \mp \widetilde{\nu}'\widetilde{y})}} \right)$$

Hereby are: L, $\widetilde{\nu}' = B'L$, $\nu' = B'\ell$, $\overline{\nu}' = B'c$, the dimensionless span and the similarity parameters of the FC with stretched flaps, of its wing and of its integrated fuselage, at lower cruising Mach number M'_∞, the quotients $\widetilde{k} = \ell / L$, $\overline{k} = c/L$, $B' = \sqrt{M'^2_\infty - 1}$ and here $\widetilde{y} = y/L$. The lift and the pitching moment coefficients of the integrated wing-fuselage FC with stretched flaps are linear and homogeneous functions with respect to the coefficients of the downwashes:

$$C_\ell \equiv 8\ell \int_{\widetilde{O}\widetilde{C}\widetilde{F}_1} \widetilde{u} \; \widetilde{x}_1 \, d\widetilde{x}_1 \, d\widetilde{y} = \ell \sum_{n=1}^{N} \sum_{j=0}^{n-1} \left(\widetilde{\Lambda}_{nj} \widetilde{w}_{n-j-1,j} + \widetilde{\widetilde{\Lambda}}_{nj} \widetilde{\widetilde{w}}_{n-j-1,j} \right) \quad ,$$

$$C_m \equiv 8\ell \int_{\widetilde{O}\widetilde{C}\widetilde{F}_1} \widetilde{u} \; \widetilde{x}_1^2 \, d\widetilde{x}_1 \, d\widetilde{y} = \ell \sum_{n=1}^{N} \sum_{j=0}^{n-1} \left(\widetilde{\Gamma}_{nj} \widetilde{w}_{n-j-1,j} + \widetilde{\widetilde{\Gamma}}_{nj} \widetilde{\widetilde{w}}_{n-j-1,j} \right) \quad . \qquad (8a,b)$$

The inviscid drag coefficients of the thin, thick-symmetrical and thick, lifting FC are:

$$C_d \equiv \ell \widetilde{C}_d = 8\ell \left[\int_{\widetilde{O}\widetilde{C}_1\widetilde{A}_1} \widetilde{u}\widetilde{w}\widetilde{x}_1 \, d\widetilde{x}_1 \, d\widetilde{y} + \int_{\widetilde{O}\widetilde{A}_1\widetilde{F}_1} \widetilde{u}\widetilde{\widetilde{w}}\widetilde{x}_1 \, d\widetilde{x}_1 \, d\widetilde{y} \right]$$

$$C_d^* \equiv \ell \widetilde{C}_d^* = 8\ell \left[\int_{\widetilde{O}\widetilde{C}_1\widetilde{A}_1} \widetilde{u}^* \; \widetilde{w}^* \; \widetilde{x}_1 \, d\widetilde{x}_1 \, d\widetilde{y} + \int_{\widetilde{O}\widetilde{C}\widetilde{C}_1} \widetilde{u}^* \; \overline{w}^* \; \widetilde{x}_1 \, d\widetilde{x}_1 \, d\widetilde{y} + \int_{\widetilde{O}\widetilde{A}_1\widetilde{F}_1} \widetilde{u}^* \; \widetilde{\widetilde{w}}^* \; \widetilde{x}_1 \, d\widetilde{x}_1 \, d\widetilde{y} \right]$$

$$C_d^{(i)} \equiv \ell \widetilde{C}_d^{(i)} = \ell \left(\widetilde{C}_d + \widetilde{C}_d^{*} \right). \tag{9a-c}$$

The optimal planform of the GO shape of FC presents an important change with respect to the chosen cruising Mach number . That is the reason that the multipoint design is realized by morphing.

3 INVISCID GLOBAL OPTIMAL SHAPE DESIGN OF FLYING CONFIGURATION WITH RETRACTED AND WITH STRETCHED FLAPS

The determination of the inviscid GO shape of the FC with movable leading edge flaps leads to two consecutive enlarged variational problems with free boundaries .

The first one concerns the determination of the GO shape of the integrated wing-fuselage configuration with retracted flaps, which is of minimum drag at higher cruising Mach number M_∞. The free parameters of the optimization are the coefficients of the downwashes w , w^* and w'^* and also the similarity parameters ν and $\bar{\nu}$ of the planforms of the wing and of the fuselage. The quotient of these similarity parameters, which depends on the purpose of the FC, is supposed to be constant.

The constraints of the inviscid GO shape's design are the following: the given lift, pitching moment and the Kutta condition along the subsonic leading edges of the thin FC component (in order to cancel the induced drag at cruise and to suppress the transversal conturnement of the flow around the subsonic leading edges, in order to increase the lift) and the given relative volumes of the wing and of the fuselage zone, the cancellation of thickness along the leading edges and the new introduced integration conditions along the junction lines between the wing and fuselage zone of the thick-symmetrical FC component (in order to avoid the detachment of the flow along these lines).

According to the optimum-optimorum theory, the GO shape of the FC is searched among the elitary FCs with the same area of their planforms which belong to the same class of FCs. The class is defined by the common properties of the elitary FCs which belong to this class. The similarity parameter ν of the planform of FC is sequentially varied and a lower limit-line of the inviscid drag functional of elitary FCs, as function of this similarity parameter ν, is obtained. For FCs with subsonic leading edges is: $0 < \nu < 1$. The position of the minimum of this limit-line gives the optimal value of the similarity parameter $\nu = \nu_{opt}$ and the corresponding elitary FC is, at the same time, the global optimized FC of the class.

The author has used its OO theory for the determination of the inviscid GO shapes of three models, namely, Adela (a wing alone) and Fadet I and Fadet II (two fully-integrated wing-fuselage FCs), which are of minimum drag , respectively, at cruising Mach numbers $M_\infty = 2;\ 2.2;\ 3$. In the (Fig. 1) is represented the GO shape of the model Fadet II.

The theoretical predicted lift, pitching moment and the pressure coefficients of the upper side of six delta FC's models (namely the wedged delta, the double wedged delta, the wedged delta wing fitted with central conical fuselage, the global optimized delta wing Adela, the global optimized and fully-integrated wing-fuselage models Fadet I and Fadet II) were measured in the trisonic wind tunnel of DLR-Köln, in the frame of research projects of the author, sponsored by the DFG.

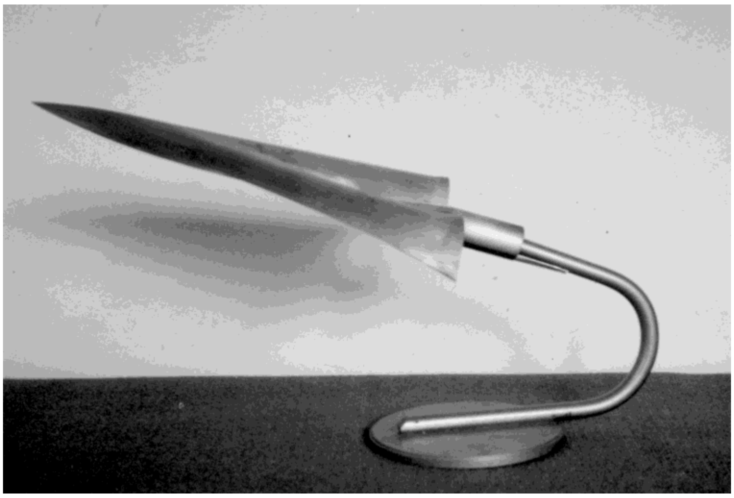
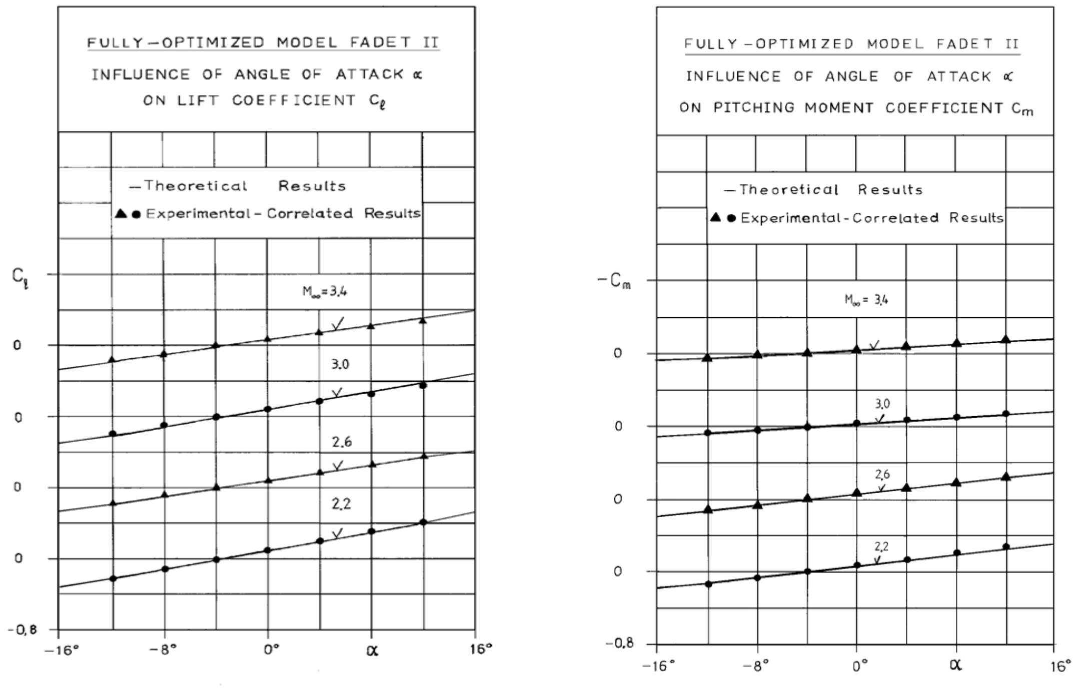
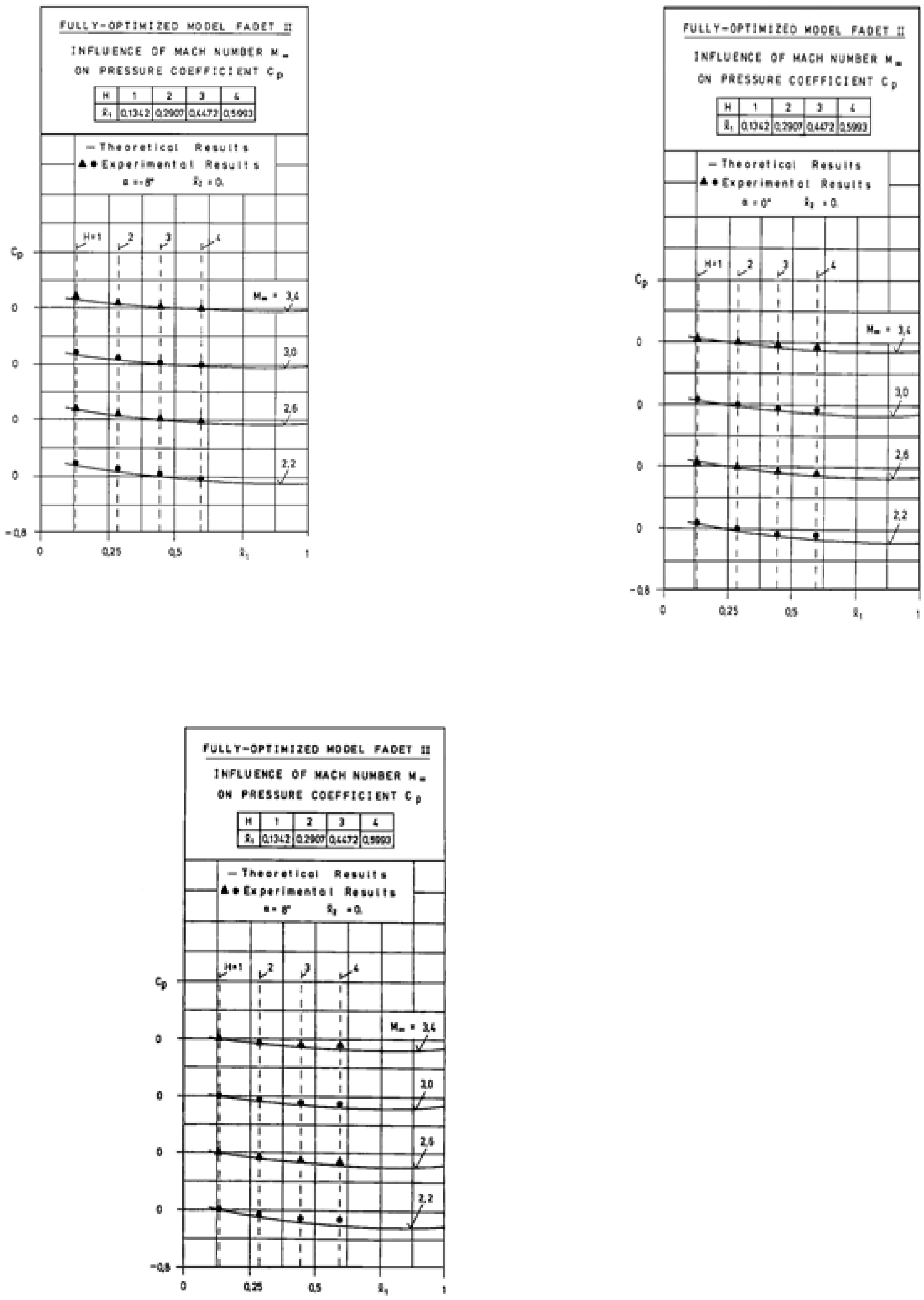


Figure 1: The Global Optimized and Fully-Integrated Shape of the Model Fadet II at Cruising Mach Number $M_\infty = 3$



Figures 2a,b: The Variation of Lift and Pitching Moment Coefficients of Model Fadet II



Figures 3a-c: The Variations of Pressure Coefficient on the Upper Side of Model Fadet II, in its Central Longitudinal Cut, for the Angles of Attack $\alpha = -8^\circ;\ 0^\circ;\ 8^\circ$

The comparisons of theoretical and experimental-correlated values of the lift and pitching moment coefficients of all these models were in very good agreements with the experimental results. In the (Figs. 2a,b), are presented these agreements between the theoretical and the experimental correlated values of the lift and pitching moment coefficients.

In (Figs. 3a-c) are represented the agreements between the theoretical and the experimental interpolated values of pressure coefficients on the upper side of the longitudinal central cuts of the model Fadet II at the angles of attack $\alpha = -8^\circ;\ 0^\circ;\ 8^\circ$.

The second new extended variational problem concerns the determination of the GO shape of the stretched flaps, in order that entire FC to be of minimum inviscid drag at the second, lower cruising Mach number $M_\infty^{'}$. The central part of the FC remains unchanged and the free parameters of optimization are now the coefficients of the downwashes $\tilde{\tilde{w}}$ and $\tilde{\tilde{w}}^*$ on the thin and thick-symmetrical components of the flaps and the similarity parameter $\nu' = B^{'}L$ of the planform of the entire FC with stretched flaps.

The constraints for the thin component are: the given lift and pitching moment coefficients and the fulfilling of Kutta condition on the subsonic leading edges of the flaps and, for the thick-symmetrical component, the given relative volume of the flaps, null-thickness along the leading edges of the flaps (sharp leading edges) and full-integration along the junction lines wing-flaps. The optimal values of the free parameters $\tilde{\tilde{w}}_{\theta\sigma}$, $\tilde{\tilde{w}}^*_{\theta\sigma}$ and ν' are obtained by cancellation of their coefficients in the first variation of the inviscid Hamilton’s operator.

For the computation of the total drag, including friction, hybrid NSL’s solutions are further proposed.

4 HYBRID SOLUTIONS FOR THE NAVIER-STOKES LAYER

The proposed hybrid solutions for the NSL use the hyperbolic potential solutions given here of the same FC, given here as outer flow at the NSL’s edge and to reinforce the NSL’s solutions. The structure of the velocity's components of the NSL are expressed as products between the corresponding potential velocity's components with polynoms with arbitrary coefficients, versus a spectral variable. These coefficients are used to satisfy the NSL's PDEs, in an arbitrary chosen number of points. Let us firstly introduce a spectral variable:

$$\eta = \frac{x_3 - Z(x_1,\ x_2)}{\delta(x_1,\ x_2)} \qquad (0 \le \eta \le 1) \tag{10}$$

The proposed forms for the hybrid numerical solutions of the velocity's components are , as in [1,2], the following:

$$u_\delta = u_e \sum_{i=1}^{N} u_i\, \eta^i \quad , \quad v_\delta = v_e \sum_{i=1}^{N} v_i\, \eta^i \quad , \quad w_\delta = w_e \sum_{i=1}^{N} w_i\, \eta^i \ . \tag{11a-c}$$

The here introduced logarithmic density function $R = \ln \rho$ and the absolute temperature T are the following:

$$R = R_w + \left(R_e - R_w \right) \sum_{i=1}^{N} r_i \, \eta^i \quad , \qquad T = T_w + \left(T_e - T_w \right) \sum_{i=1}^{N} t_i \, \eta^i \quad . \tag{12a,b}$$

The pressure p is computed by using the physical equation of perfect gas and, for the viscosity μ, an exponential law is used:

$$p = R_g \, \rho \, T = R_g \, e^R \, T \quad , \qquad \mu = \mu_\infty \left(\frac{T}{T_\infty} \right)^{n_1} \quad . \tag{13a,b}$$

The free coefficients $u_i,\ v_i,\ w_i,\ r_i$ and t_i are used to satisfy the NSL's PDEs in some chosen points. If the hybrid forms for the velocity's components (11a-c) are introduced in the continuity's PDE and the collocation method is used, the coefficients r_i are determined only as functions of the coefficients of the velocity's components, by solving a linear algebraic system and the coefficients t_i satisfy the PDE of absolute temperature and are also obtained only as functions of the coefficients of the velocity's components by solving of a transcendental algebraic system. A splitting of the NSL's PDEs is obtained and the physical entities are expressed only as function of the spectral coefficients of the velocities components and can be easy updated in an iterative process. A speed up of computation time is obtained. The coefficients of velocity's components are determined by using the impulse PDEs, which are iteratively solved.

The hybrid solutions for the NSL presented here are reinforced numerical solutions, which present important analytical properties, namely: they have correct last behaviors, they have correct jumps due to the singularities located only along the singular lines (like junction lines wing-fuselage, subsonic leading edges of the wing with retracted flaps and junction lines wing-fuselage, junction lines wing-flaps and subsonic leading edges of the flaps of the FC with stretched flaps) obtained according to the principle of minimal singularities which fulfill the jumps and the singularities are balanced, they are accurate because the partial-derivatives of velocity's components can be exactly computed, they are split due to the use of the logarithmic density function and therefore they produce a speed up of the computation time, they fulfill automatically the non-slip condition on the FCs surface, they are matched with the outer potential flow and for moderate perturbations, they are reduced to the potential solutions at the NSL's edge, if some boundary conditions are satisfied and they do not need interface. Additionally, for hyperbolic PDEs the boundary condition on its characteristic surface is automatically fulfilled . The hybrid solutions of the NSL's PDEs are used for the computation of the friction drag coefficient of the FC. The skin friction coefficient at the wall is:

$$\tau_{x_1}^{(w)} \equiv \tau_{x_1} \Big|_{\eta = 0} = \mu_f \left. \frac{\partial \, u_\delta}{\partial \, \eta} \right|_{\eta = 0} = \mu_f u_1 u_e \quad . \tag{14}$$

The friction drag coefficient and the total drag of the FC, with arbitrary camber, twist and thickness distributions are:

$$C_d^{(f)} = 8 \ \nu_f u_1 \int_{\widetilde{O}\widetilde{A}_1\widetilde{C}} u_e \widetilde{x}_1 \, d\widetilde{x}_1 \, d\widetilde{y} \ , \qquad C_d^{(t)} = C_d^{(f)} + C_d^{(i)} \quad . \tag{15a,b}$$

These hybrid NSL's solutions are also used for the viscous design of the GO shape of FC.

5 ITERATIVE OPTIMUM-OPTIMORUM THEORY

The viscous iterative OO theory of the author is proposed for the viscous determination of the GO shape of the FC with flaps in retracted position in order to present a total minimum drag at the first higher cruising Mach number and also for the viscous determination of the GO shape of the flaps of the FC with stretched flaps, in order to obtain a minimum total drag for the entire wing-fuselage FC with the flaps in stretched position, at the second lower Mach number. The viscous iterative OO theory uses the inviscid hyperbolic potential solutions as start solutions and the inviscid GO shape of these FCs as surrogate models, **only** in its first step of iteration. An intermediate computational checking of this inviscid GO shape of the FC is made with own hybrid solvers, for the three-dimensional compressible NSL. The friction drag coefficient $C_d^{(f)}$ of the FC is computed and the inviscid GO shape is checked also for the structure point of view. A weak interaction aerodynamics-structure is proposed. Additional or modified constraints, introduced in order to control the camber, twist and thickness distributions of the GO shape, for structure reasons, are here proposed. In the second step of optimization, the predicted inviscid GO shape of the FC is corrected by including these additional constraints in the variational problem and of the friction drag coefficient in the drag functional. The chart flow of the iterative OO theory is given in the (Fig. 4).

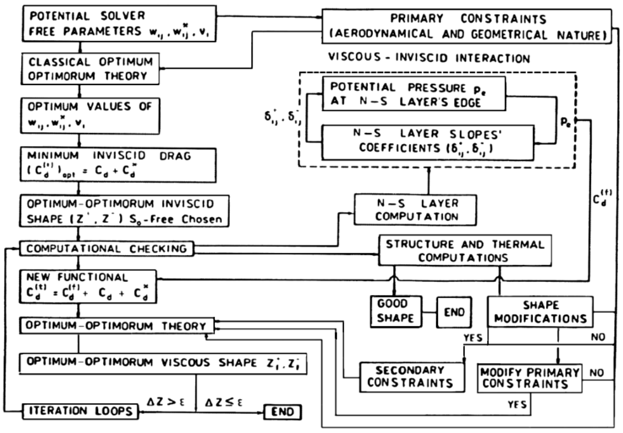


Figure 4: The Iterative Optimum-Optimorum Theory

6 CONCLUSIONS

- The author proposes analytical three-dimensional hyperbolic potential solutions for the computation of the axial disturbance velocity over the wing-fuselage FC fitted with flaps in retracted and in stretched positions, which are useful for the computation of its lift, pitching moment and pressure coefficients of the FC. These solutions are given in integrated forms, are in good agreement with experimental results and are used as start solutions for the determination of the inviscid GO shapes of FC with retracted and with stretched flaps. These potential solutions are the start solutions for the determination of the inviscid GO shape of the FC.

- The hybrid numerical solutions for the three-dimensional full PDEs of NSL, proposed here, use these hyperbolic potential solutions twice, namely: as outer flow at the NSL's edge and for the analytical hybridization of the numerical solutions. The velocity's components are expressed as products of potential solutions and polynoms with arbitrary coefficients, which are used to satisfy the NSL's PDEs in some chosen points. By using a logarithmic density a splitting of NSL's PDEs is realized.
- The hybrid numerical solutions of the NSL, proposed here, are accurate because they are meshless, their derivatives can be easy and exact computed, they automatically fulfill the non-slip condition and have important analytical properties given above, due to the analytical hybridization and due to the splitting. Artificial viscosity and correction coefficients are not used.
- The evolutive iterative optimum-optimorum theory is a special deterministic strategy, developed by the author, for the solving of the enlarged variational problem with free boundaries. The GO shape of FC is searched inside of a given class of elitary FCs. Therefore it has almost all attributes of genetic algorithms like: migrations (in the drag functional and in the constraints in the early steps of its iteration), crossover (by the construction of its NSL start solutions up the second step of its iterations), mutation (in the start solutions and in the constraints), multiple selections (inside of a class and among different classes of elitary FCs).
- It allows the performing of the multipoint global optimal design by morphing. Movable leading edge flaps are used for this purpose and the both enlarged variational problems with free boundaries, which occur, are solved.
- It allows the performing of the multidisciplinary global optimization. A weak interaction aerodynamics-structure is proposed, via additional or modified constraints requested for the structure purpose.
- It is flexible because it can use different start solutions, drag functionals and constraints, which can be changed in the early steps of iterations.
- It is economic and competitive, due to the splitting of the NSL's PDEs and of analytical hybridization a speed up of computation time occurs.

The morphing of FCs, by using spanwise movable leading edge flaps, is especially useful for supersonic aircraft of small size, like business and rescue jets, which must be able to take off and to land on shorter runways and for UAVs.

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